Concrete / Pictorial / Abstract Maths Calculation Guidance

This guidance has been largely adapted from the White Rose Maths Hub Calculation Guidance with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils across the school, it is important that that the mathematical language used in maths lessons and across the curriculum reflects the vocabulary used throughout this guidance.

THE WEALD
CofE PRIMARY SCHOOL

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MathsHUBS

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

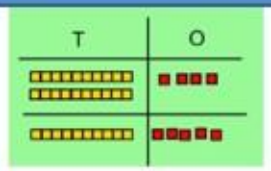
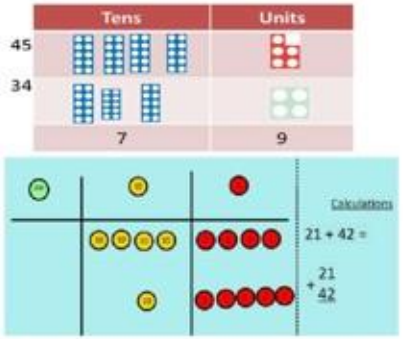
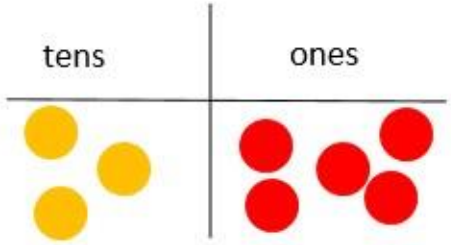
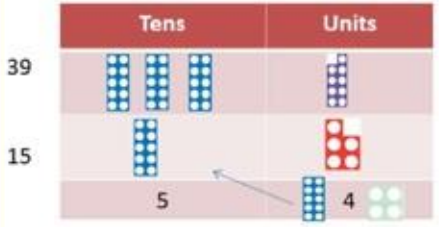
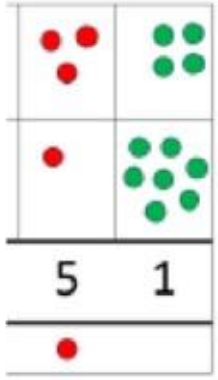
Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians are being extended. An extension tasks to deepen understanding is the most simplistic way around this.

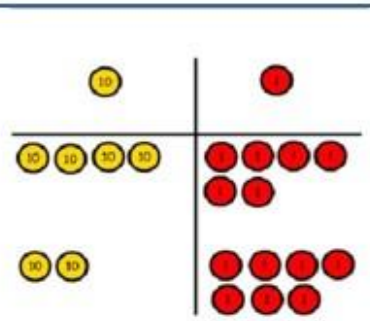
Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and provide a common language communicate cognitive models for (2015)	*Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951) to allow children to make connections. Morgan, D.*	*Real things and structured images enables children to understand the abstract. The concrete and the thinkers. They can also images are a means for children to understand the with which to symbolic so it's important to move between all modes abstract ideas. Drury, H. (2016)*

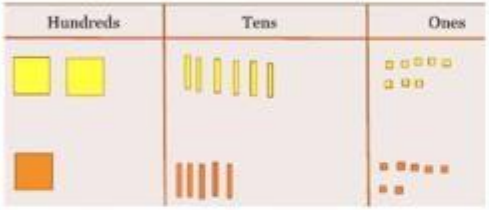
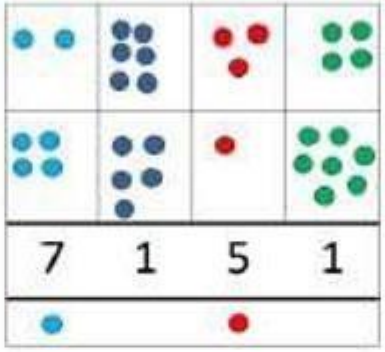
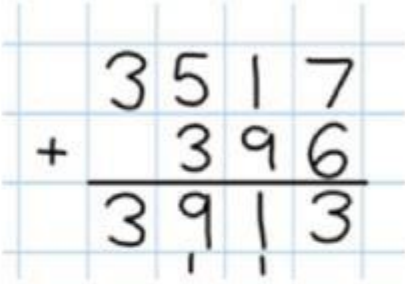
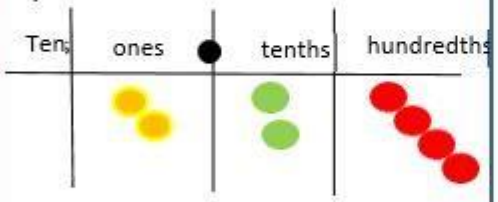
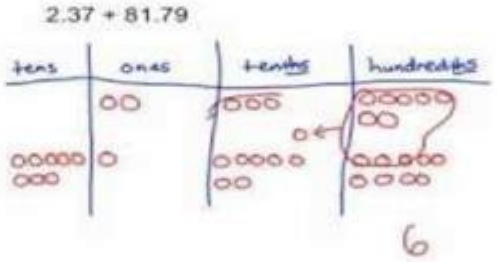
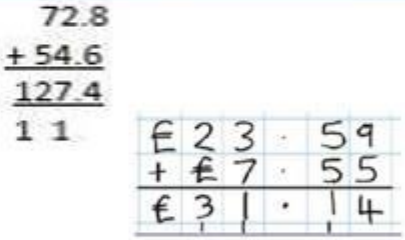
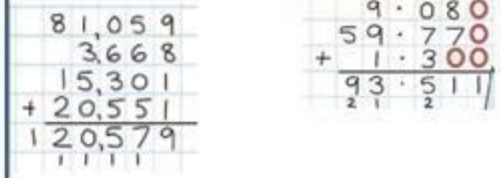
The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

YEAR 3 Addition

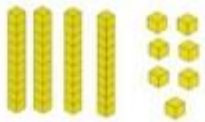
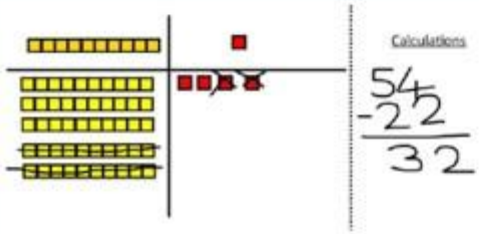
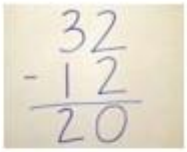
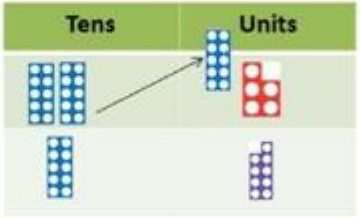
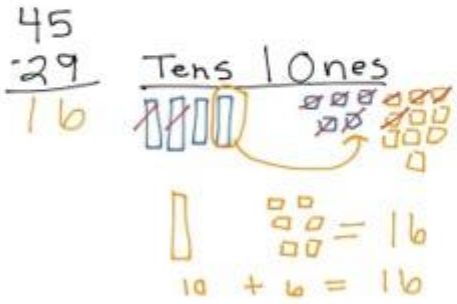
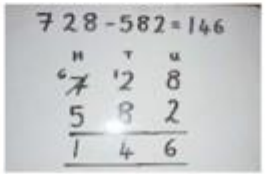
Objective /Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.	 Dienes or numicon Add together the ones first, then the tens.  Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and place value counters.	 3 4 +1 7 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1

	 46 + 27 = 73		
Estimate the answers to questions and use inverse operations to check answers	 Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation. 	Building up known facts and using them to illustrate the inverse and to check answers: 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

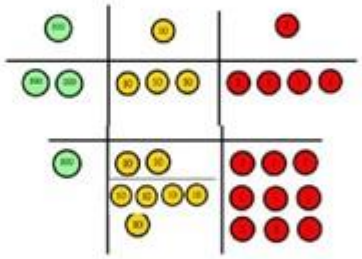
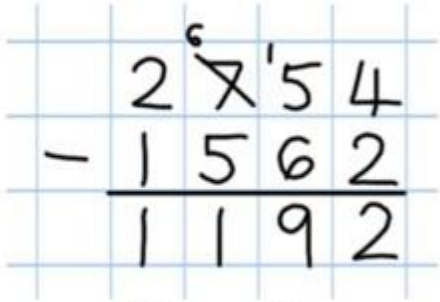
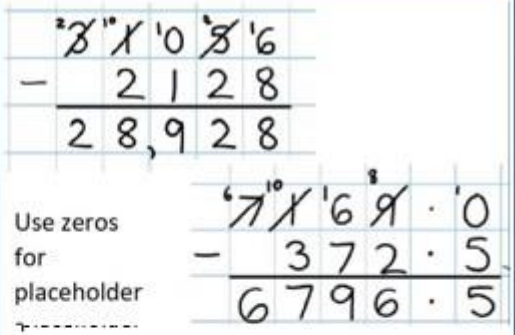
YEARS 4 – 6 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Years 4 – 6 Estimate and use inverse operations to check answers to a calculation	**AS per Year 3**		
Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		
Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders. 

YEAR 3 - SUBTRACTION

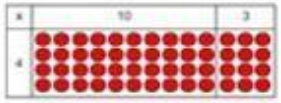
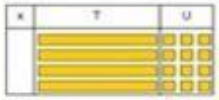
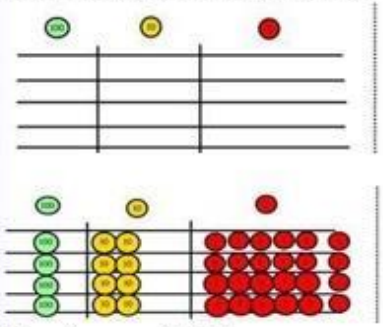
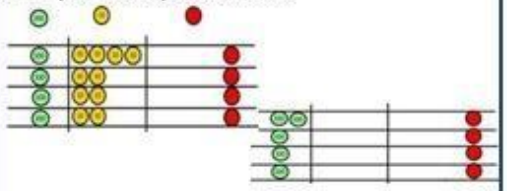
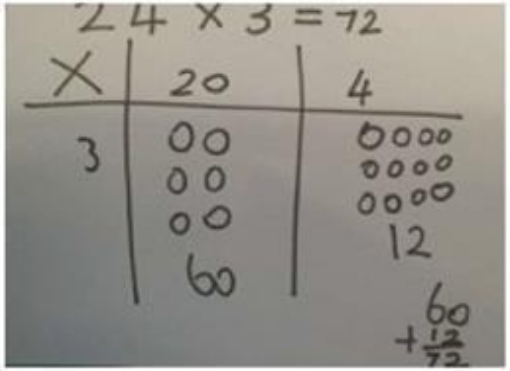
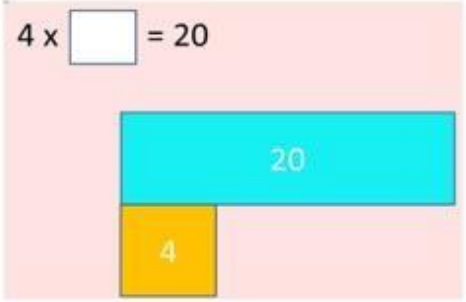
Objective/ Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	  Use base 10 or Numicon to model	 Draw representations to support understanding	47 – 24 = 23 40 + 7 – 20 + 4 20 + 3 Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

YEARS 4 – 6 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange 10 x2 = 20 6 x2 = 12
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholder s.
Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	

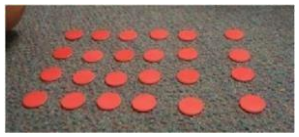
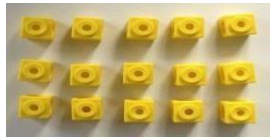
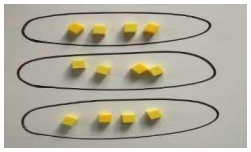
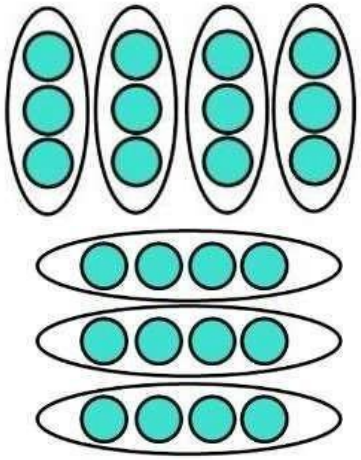
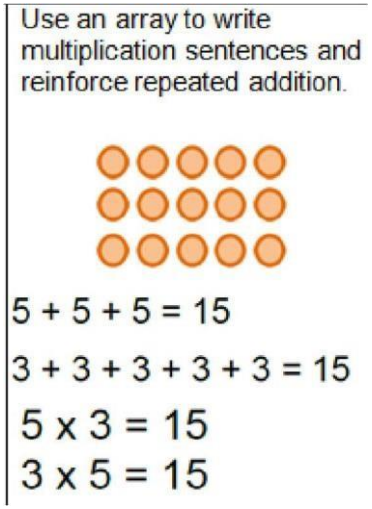
YEAR 3 MULTIPLICATION

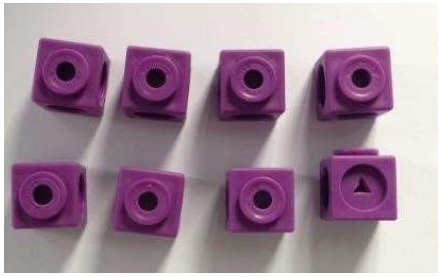
Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

Objective /Strategy	Concrete	Pictorial	Abstract
Grid method, progressing to the formal method Multiply 2 digit numbers by 1 digit numbers	Show the links with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Calculations 4 x 126 Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  24 x 3 = 72 Bar model are used to explore missing numbers  4 x ☐ = 20 20 4	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | **210 + 35 = 245** Move forward to the formal written method: 35 X 7 245 3

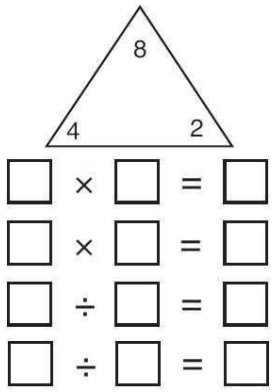
Solve problems, including missing number problems, integer scaling problems,			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats, how many different outfits?

Objective / Strategy	Concrete	Pictorial	Abstract

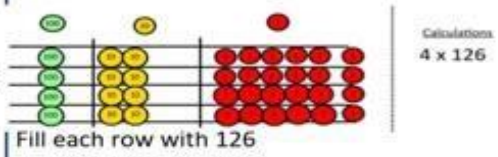
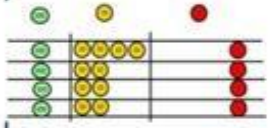
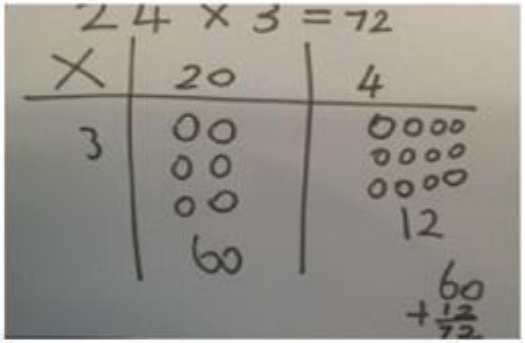
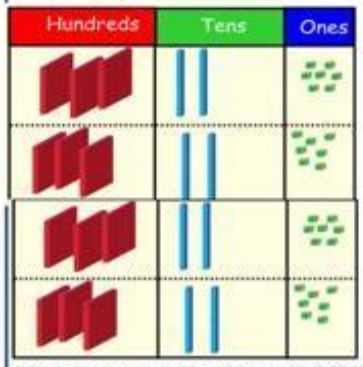
<table>
<tr>
<td>Multiplication is commutative</td>
<td>Create
 
arrays using
counters
and cubes
and
Numicon.
  
Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.
   </td>
<td>Use representations of arrays to show different calculations and explore commutativity.
  </td>
<td>12 = 3 × 4 12 = 4 ×
3
 
</td>
</tr>
<tr>
<td>Using the Inverse
This should be
taught alongside

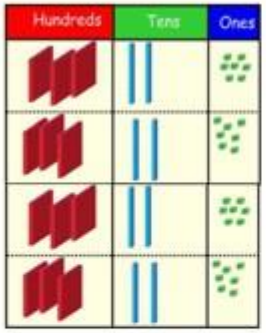
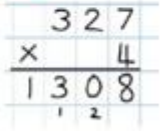
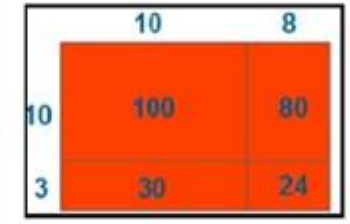
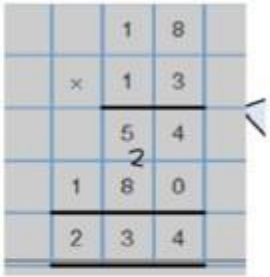
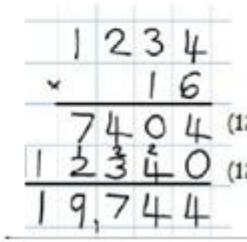
division, so pupils</td>
<td>  </td>
<td></td>
<td>2 x 4 = 8
4 x 2 = 8
8 ÷ 2 = 4

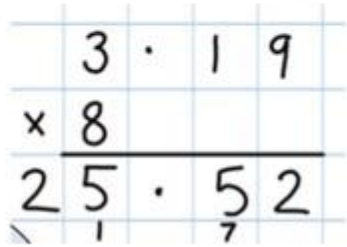
8 ÷ 4 = 2</td>
</tr>
</table>

learn how they *work alongside each other.*			8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

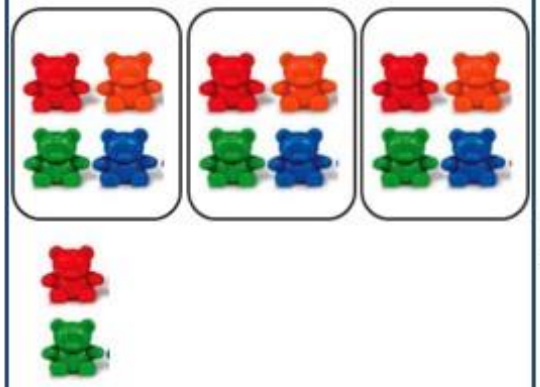
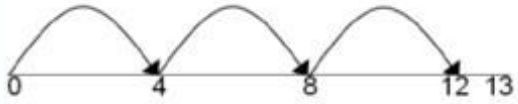
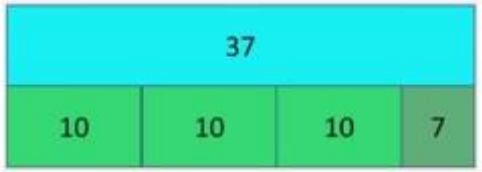
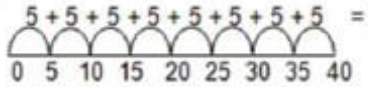
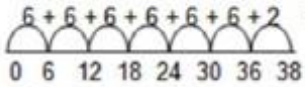
YEARS 4 – 6 Multiplication

Objective /Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126   Add up each colu…nes making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | x | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	| x | 300 | 20 | 7 | | --- | --- | --- | --- | | 4 | 1200 | 80 | 28 | The grid method my be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 |	3	2	7 | | --- | --- | --- | --- | | x | | | 4 | | 1 | 3 | 0 | 8 | | | 1 | 2 | | This may lead to a compact method.

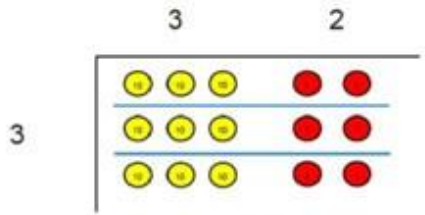
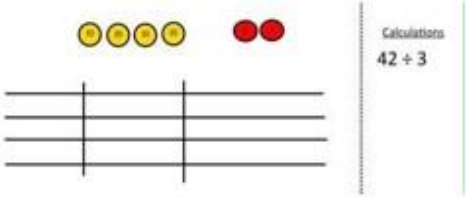
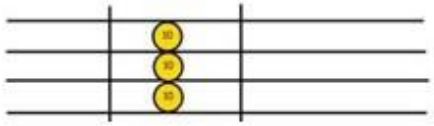
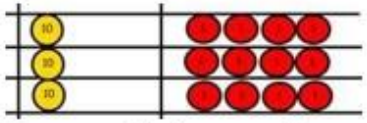
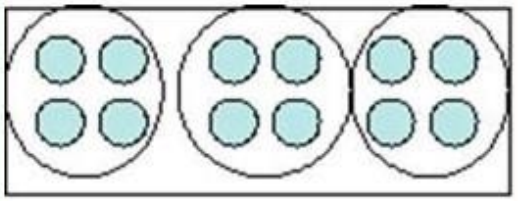
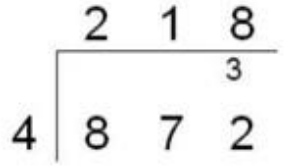
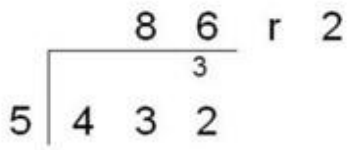
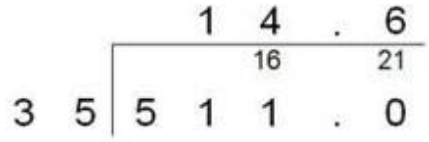
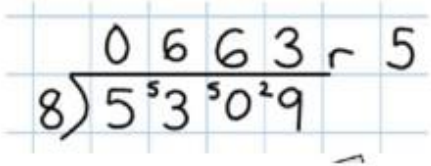
Objective /Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	 It is important at this stage that they always Multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642		327 x 4 28 80 120 1308 
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row.  Show multiplying by 10 by putting zero in units first

Objective/Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

YEAR 3 Division

Objective/Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  remainder: 5s in 40?"  mainder:  rs, when it becomes inefficient to count in single mu orded using known facts.	Complete written divisions and show the remainder using r. 

Year 4-6 Division

Objective/Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3   Calculations 42 ÷ 3 Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Long Division

Step 1—a remainder in the ones

```
    h  t  o
      0  4  1 R1
   ---------
4 ) 1  6  5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th  h  t  o
        0  4  0  0 R7
   ------------
8 ) 3  2  0  7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

```
    h t o
    0 6 1
  ---------
4 ) 2 4 7
    - 4
    -----
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
     th h t o
     0 4 0 2
  -----------
4 ) 1 6 0 9
      - 8
      -----
        1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 -2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0 There are no more digits to drop down. The quotient is 139.